Contributions of the direct memory retrieval and the quantitative operation to the simple mental addition, subtraction and multiplication

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暗算（加算、減算、乗算）における記憶検索及び量的操作処理の役割

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要約

本研究の目的は暗算（加算、減算、乗算）における記憶検索及び量的操作処理の役割について検討することであった。暗算（加算、減算、乗算）を一次課題、母音判断課題及び方向判断課題を二次課題とする 2 重課題手続きが用いられた。母音判断課題は記憶検索処理、方向判断課題は量的操作処理をそれぞれ干渉すると考えられる。 実験の結果、乗算においては方向判断課題より母音判断課題の方が干渉効果が大きいこと、 一方、加算においては母音判断課題より方向判断課題の方が干渉効果が大きいことが明らかになった。 減算では母音判断課題と方向判断課題の干渉効果の間に差が見られなかった。これらの結果は、乗算においては記憶検索処理が、加算においては量的操作処理が重要であることを示唆している。また、減算においては記憶検索処理及び量的操作処理の両方がなされている可能性がある。

Key words

simple arithmetic, direct memory retrieval, quantitative operation, dual-task

1. Introduction

How can we do simple mental arithmetic such as the addition (e.g., 2 + 3), the subtraction (e.g., 3 - 2), and the multiplication (e.g., 2 X 3)? In spite of apparent simplicity of such arithmetic problems, this question has been discussed for a long time in cognitive psychology, and the conclusive answer has not been drew yet (e.g., Ashcraft, 1992; 1995).

Dehaene (1992) proposed a general framework of number processing, called the triple code model (see Dehaene & Cohen, 1995). In this model, three types of internal representations for the number processing are assumed: visual number form, verbal word frame, and analog magnitude representation. Both the visual number form and the verbal word frame are notation-dependent representations, which are representations at the identification level for the visual numbers (e.g., Arabic and written-word numbers) and the spoken numbers, respectively. The analog magnitude representation is a notation-independent quantitative semantic representation of numbers, and assumed to be a left-to-right, compressed, analog number line (Dehaene, Bossini, & Giraux, 1993; Dehaene, Dupoux, & Mehler 1990; Reynvoet & Brysbaert, 1999).

According to the triple code model, the simple mental arithmetic is based on the two different mental processing, the direct memory retrieval and the quantitative operation (Dehaene & Cohen, 1995). The critical difference between these two types of mental processing is whether the analog magnitude representation mediates the mental arithmetic (i.e., quantitative operation) or does not (i.e., direct memory retrieval). The direct memory retrieval refers to the mental processing where the arithmetic problem (e.g., 3 X 7) is translated into the verbal word frame, and then the arithmetic facts (e.g., multiplication table) are directly retrieved from the long-term memory (Ashcraft, 1992; Dehaene & Cohen, 1997). Deheane argued that the verbal word frame is the obligatory entry code for accessing the arithmetic facts, and the direct memory retrieval does not require the quantitative processing of operands (e.g., Dehaene & Cohen, 1995).

On the other hand, the quantitative operation is based on the quantitative processing of operands. Some types of the quantitative operation have been proposed (e.g., Dehaene & Cohen, 1995; Restle, 1970). For instance, Restle (1970) proposed that the mental addition depends an analog quantitative manipulation on the mental analog number line (i.e., analog magnitude representation in Dehaene's model). According to Restle, both addends are translated into analog quantities, and then these are combined mentally, resulting in that the combined quantity denotes the sum. Moreover, Deheane and Cohen (1995) proposed a semantic elaboration as one of the quantitative operations. According to Deheane and Cohen (1995), subjects recode the arithmetic problem (e.g., 9 + 5 [original problem] = 9 + 1 + 4 [recoded problem]), and then, retrieve the arithmetic facts from the long-term memory (e.g., 9 + 1 = 10, 10 + 4 = 14). Dehaene and Cohen (1995) argued that the former processing (i.e., semantic elaboration) was based on the quantitative processing of numbers. For example, subjects should notice that 9 is close to 10 before recoding the problem. According to Dehaene's triple code model, the quantitative operation such as an analog quantity manipulation (Restle, 1970) and a semantic elaboration (Deheane & Cohen, 1995) are based on the analog magnitude representation.

How the two types of processing (i.e., direct memory retrieval and the quantitative operation) contribute to the mental addition, subtraction and multiplication? Findings form studies with normal subjects have indicated that the mental multiplication entirely depends on the direct memory retrieval (see Ashcraft, 1992; 1995). For the mental addition, however, the findings are less consistent among studies (Stazyk, Ashcraft, & Hamann, 1982; Siegler, 1989). As a result, some researchers propose that the mental addition also entirely depends on the direct memory retrieval (e.g., Miller, Perlmutter, & Keating, 1984), whereas others propose that the quantitative operation (e.g., semantic elaboration) contributes to the mental addition (e.g., LeFevre, Sadesky & Bisanz, 1996). For the mental subtraction, available evidence is fewer than the multiplication and addition (but see Woods, Resnick, & Groen, 1975).

Findings from neuropsychological studies also give suggestions to this issue (e.g., Dehaene & Cohen, 1995; Dehaene, Tzourio, Frak, Raynaud, Cohen, Mehler, & Mazoyer, 1996). For example, Deahene and Cohen (1997) reported the two patients with numerical disorder, MAR and BOO, who were suffered from brain damage in different foci (right inferior parietal lesion or left subcortical lesion, respectively). They examined their abilities of the simple mental arithmetic and the quantitative processing of number itself (e.g., numerical size judgment). Likewise the quantitative operation for the simple mental arithmetic, it is assumed that the quantitative processing of number such as numerical size judgement is based on the analog magnitude representation (e.g., Dehaene, 1989; Moyer & Landauer, 1967). Deahene and Cohen (1997) revealed that, for MAR, the mental subtraction and the quantitative processing of number were impaired, while the mental addition and multiplication were relatively preserved. On the other hand, for BOO, the mental multiplication was impaired, while the mental addition and subtraction, and the quantitative processing of number were relatively preserved. But, BOO's the reaction time (RT) of the addition was very slow, and it was increased with the size of smaller addends, suggesting that the complementary strategy (e.g., serial counting) might be used only in the mental addition. Based on the findings, Deahene and Cohen (1997) concluded that the mental multiplication and addition depend on the direct memory retrieval which is mediated by the verbal word frame, whereas the mental subtraction depends on the quantitative operation which is based on the analog magnitude representation.

Furthermore, Cohen and Dehaene (2000) reported a patient with numerical disorder, VOL, who was suffered from focal brain damage (left posterior lesion). They also investigated her abilities of the simple mental arithmetic and the quantitative processing of number (e.g., numerical size judgment), and showed that the mental addition and subtraction, and the quantitative processing of number were relatively preserved, whereas the mental multiplication was impaired. Hence, Cohen and Dehaene (2000) concluded that the mental multiplication depends on the direct memory retrieval, whereas the mental subtraction and the addition depend on the quantitative operation.

In sum, the findings of cognitive psychological and neuropsychological studies have indicate that the mental multiplication exclusively depends on the direct memory retrieval. In comparison with the multiplication, however, it is less clear how the direct memory retrieval and the quantitative operation contribute to the simple mental addition and subtraction. Some previous findings suggest that both the direct memory retrieval and the quantitative operation might be based on the mental addition, and the contribution of quantitative operation might be larger in the addition than in the multiplication (e.g., Cohen & Dehaene, 2000; LeFevre, Sadesky & Bisanz, 1996). However, available evidences about the mental addition are something ambiguous. According to neuropsychological data, it is assumed that the mental subtraction might depend more on the quantitative operation than the direct memory retrieval. However, experimental evidences are very limited to conclude this possibility. In the present study, therefore, we examined how the direct memory retrieval and the quantitative operation contribute to the mental addition, subtraction, and multiplication.

For this aim, the dual-task procedure seemed to be useful (Hecht, 2002; Noel, Desert, Aubrun & Seron, 2001; Seitz & Schumann-Hengsteler, 2000). Recently, Lee and Kang (2002) investigated the simple mental subtraction and multiplication using the dual-task procedure. They asked the subjects to solve the simple subtraction and multiplication problems in the two dual-task conditions: the verbal and the non-verbal secondary tasks. In the verbal task condition, subjects were required to whisper a nonword string during solving the subtraction or multiplication problems repeatedly. In the non-verbal task condition, subjects were required to memorize the shape and location of an abstract figure before the solving the subtraction or multiplication problems, and, after the calculation, subjects were asked to select the memorized figure among four abstract figures (1 target and 3 distractors; i.e., visuo-spatial recognition memory test). Lee and Kang (2002) found that the verbal task interfered with the mental multiplication but not the mental subtraction, whereas the non-verbal task interfered with the mental subtraction but not the mental multiplication. Lee and Kang (2002) argued that their findings could be interpreted by Dehaene's triple code model. According to Lee and Kang (2002), their verbal task interfered with the translation processing from the arithmetic problems to the verbal word frame, resulting in that the direct memory retrieval was disturbed. Hence, their result, the verbal task interfered with the mental multiplication, indicate that the mental multiplication might depend on the direct memory retrieval. On the contrary, Lee and Kang (2002) argued that their non-verbal task interfered with translation from the operands in arithmetic problems to the analog magnitude representation, resulting in that the quantitative operation was disturbed. Thus, their finding, the non-verbal task interfered with the mental subtraction, indicated that the mental subtraction might depend on the quantitative operation.

The findings of Lee and Kang (2002) are consistent with the previous findings (Ashcraft, 1992; Deahene & Cohen, 1997), and suggest that the dual-task procedure is advantageous for addressing the issue of the present study. However, two problems in Lee and Kang (2002) must be excluded. The first problem is that the subprocesses were different between the non-verbal task and the verbal task. For example, the verbal task includes the information maintaining process with covert motor response (i.e., whisper), whereas the non-verbal task includes the information maintaining process without covert motor response. Moreover, the subprocesses for the recognition memory test (e.g., verification, decision and production of motor response) were included in the non-verbal task but not in the verbal task. Because the sub-processes are very different between the two secondary tasks, the direct comparison between the verbal and the non-verbal task conditions is not desirable. That is, it is not clear whether the obtained results (interference effects) reflect on the difference in the quality (i.e., verbal or non-verbal) or the subprocess included in the secondary tasks.

Moreover, Lee and Kang (2002) claimed that their non-verbal task would interfere with the quantitative operation which was based on the analog magnitude representation. In fact, ample evidence have showed that the quantitative processing of numbers associate with visuo-spatial processing, suggesting that the visuo-spatial processing could disturb the quantitative operation (see Butterworth, 1999; Dehaene, 1997). However, Fias, Lauwereyns, and Lammertyn (2001) examined the relationship between the quantitative processing of numbers and the visuo-spatial processing, and found that some types of visuo-spatial processing (e.g., object orientation judgment) associated with the quantitative processing of numbers, but not with the other types (e.g., color or shape judgment). Hence, second problem of Lee and Kang (2002) is that their non-verbal task was not constructed to disturb the quantitative operation more directly on the evidences from the previous studies.

In the present study, therefore, we excluded these two problems of Lee and Kang (2002), and examined how the direct memory retrieval and the quantitative operation contribute to the mental addition, subtraction, and multiplication.

2. Experiment 1

In the present study, we employed two secondary tasks: the vowel and the orientation recognition tasks. These two tasks were assumed to include similar subprocesses such as the identification, the information maintaining without covert motor response, the decision, and the production of motor (hand) response. Therefore, we assumed that the obtained results depend on the difference of quality between the secondary tasks rather than that of subprocess included.

As mentioned above, behavioral data of Fias et al. (2001) indicated that the orientation processing associated with the quantitative processing of numbers. Moreover, Rushworth, Nixon and Passingham (1997) showed that the orientation processing is supported by the inferior parietal lobe. The neuropsychological studies have revealed that the quantitative processing of numbers are also based on the inferior parietal lobe, suggesting that the similar brain area associate with the mental processing of object orientation and numerical quantitative (Dehaene, Spelke, Pinel, Stanescu & Tsivkin, 1999; Pinel, Le Clec'H, van de Moortele, Naccache, Le Bihan, & Dehaene, 1999). That is, the previous evidences indicated that the orientation recognition task could disturb the quantitative operation which is based on the analog magnitude representation.

2.1 Method

2.1.1 Subjects

Thirty six undergraduate students of Nagoya University and Aichi Shukutoku University (30 males and 6 females) participated in the experiment for course credit. The average age of subjects was 19.7 years (*SD* = 0.7 years). All subjects had normal or corrected-to-normal vision, and were unaware of the purpose of the study.

2.1.2 Stimuli

By using all single digits except for 0 and 1, we created 36 number pairs in which tie pairs (e.g., 3 3) were not included. By using the 36 number pairs, for each operation type (addition, subtraction or multiplication), we constructed three lists in which half of the first operands (i.e., 18 pairs) were larger than the second operands (e.g., 5 2). This resulted in creating the total 9 lists. One of three lists was assigned for each dual-task condition (i.e., vowel recognition, orientation recognition, and control [i.e., mental arithmetic only]). The assignment of a list to the dual-task condition was counterbalanced between the subjects.

For the vowel and the orientation recognition tasks, two syllables (/Ka/ and /Ko/) and two types of rectangles (vertical and horizontal oriented rectangles) were prepared.

2.1.3 Design

The variables manipulated were the operation type (addition, subtraction, multiplication) and the dual-task (vowel recognition, orientation recognition, control). Both variables were manipulated within a subject.

2.1.4 Procedure

Each subject was individually tested. Operation type (addition, subtraction, and multiplication) was changed after three sessions for the dual task condition, and the order of operation type was counterbalanced between the subjects. In each operation, subjects took part into three sessions (vowel recognition, orientation recognition, and control) in which 36 number pairs were presented. The order of 36 number pairs in each session was completely randomized for each subject. In the vowel recognition task, in each trial, subjects were asked to memorize a syllable (i.e., /Ka/ or /Ko/) which was presented simultaneously with a number pair for a brief time (200ms), and to judge whether the vowel of the syllable had been /a/ or /o/ after solving an arithmetic problem. In the orientation recognition task, in each trial, subjects were asked to memorized a rectangle (i.e., vertical or horizontal orientation) which was presented simultaneously with a number pair for a brief time (200ms), and to judge whether the orientation of the rectangle had been vertical or horizontal after solving an arithmetic problem. In the control condition, subjects were required just to solve arithmetic problems. The order of three sessions (vowel recognition, orientation recognition, and control) was also counterbalanced between the subjects.

This experiment was run on an Apple Performa 6310, controlled by the time schedule of Psyscope (Cohen, MacWinney, Flatt, & Provost, 1993). Responses were recorded by the computer keyboard. For all operations, the procedure in each dual-task condition was identical, except for the task requirement (i.e., operation type). In the control condition, first, a fixation stimulus appeared at the center of monitor for 500 ms. Following a inter-stimulus-interval for 500 ms, a number pair (e.g., 4 6) emerged at the center of monitor until subjects typed in the answer using number keypad of a keyboard with the index finger of the right hand. The inter-trial-interval was 1,000 ms. According to Lee and Kang (2002), RTs of mental arithmetic were measured from the onset of number pair to the first keystroke made with an answer. In the vowel recognition task, first, a fixation stimulus appeared at the center of monitor for 500 ms. Following a inter-stimulus-interval for 500 ms, a number pair (e.g., 4 6) and a syllable (/Ka/ or /Ko/) were presented. In the half of trials in a session, /Ka/s were presented, whereas /Ko/s were presented in the other half of trials. The order of the syllables presented was completely randomized for each subject. The presentation location of number pair was identical to the control condition. A syllable was presented bellow the number pair, and it was vanished after 200 ms. A number pair was shown until subjects typed in the answer using number keypad of a keyboard with the index finger of the right hand. After subjects' typing answer, the instruction message for the vowel recognition task was appeared. For the vowel recognition task, the subjects were instructed to memorize a syllable and to judge whether vowel of the syllable had been /a/ or /o/ following the instruction message by pushing the assigned keys (Z or X key) with the left hand. The inter-trial-interval was 1,000 ms. In the orientation recognition task, first, a fixation stimulus appeared at the center of monitor for 500 ms. Following a inter-stimulus-interval for 500 ms, a number pair (e.g., 4 6) and a rectangle (vertical or horizontal) were presented. In the half of trials in a session, vertical rectangles were presented, whereas horizontal rectangles were presented in the other half of trials. The order of the rectangles presented was completely randomized for each subject. The presentation location of number pair was identical to the control condition. A rectangle was presented bellow the number pair and it was also vanished after 200 ms. A number pair was shown until subjects typed the answer using number keypad of a keyboard with the index finger of the right hand. After subjects' typing answer, an instruction message for the orientation decision task was appeared. For the orientation recognition task, the subjects were instructed to memorize a rectangle and to judge whether orientation of rectangle had been vertical or horizontal following the instruction message by pushing the assigned keys (Z or X key) with left hand. Inter-trial-interval was 1,000 ms.

Training session, which consisted of 15 trials, was given before each session. Short resting periods (about 1 minute) were inserted between the operations. Subjects were seated 60 cm away from the monitor. The entire duration of the experiment was about 50 min.

3. Results

The average error rate in each operation was 3.9% (addition), 3.2% (subtraction), and 4.1% (multiplication) in the control condition, 4.6% (addition), 4.4% (subtraction), and 4.9% (multiplication) in the vowel recognition task, 4.7% (addition), 4.6% (subtraction), and 4.3% (multiplication) in the orientation recognition task.

Trials which deviated 2 *SD* from the mean RT of correct response were excluded in the analysis. For the vowel and the orientation recognition conditions, we computed mean RT of trials in which both responses of the mental arithmetic and the secondary task were correct (Table 1). For the control condition, we computed mean RT of trials in which response of the mental arithmetic was correct (Table 1).

Table 1: Mean RT (and SE) of correct responses on addition, subtraction and multiplication in each secondary task condition

	Control	Vowel judgment	Orientation Judgment
Addition	899 (26.1)	1096 (34.5)	1128 (40.5)
Subtraction	1010 (28.1)	1350 (46.7)	1361 (52.7)
Multiplication	1016 (28.5)	1293 (42.5)	1221 (41.9)

To evaluate the interference effects of the vowel and the orientation recognition task on the mental arithmetic, we subtracted mean RT of the control condition from that of each secondary task condition for each subject. And, a 3 (operation: addition, subtraction, multiplication) X 2 (dual-task: vowel recognition, orientation recognition) ANOVA was conducted for the subtracted RT (Figure 1). The main effect of operation was significant ($F(2, 70) = 11.41$, $MSe = 345786.29$, $p<.001$). The interference effect of dual-task was larger in the subtraction (345ms) than in the addition (214ms) and the multiplication (241ms) (Tukey's $HSD = 69.76$). The main effect was dual-task was not significant ($F(1, 35) = 0.20$, $p>.6$). More importantly, the operation X dual-task interaction was significant ($F(2, 70) = 6.84$, $MSe = 54537.58$, $p<.01$; see Figure 1). Simple main effect test showed that the effect of dual-task was significant in the multiplication ($F(1, 105) = 16.84$, $MSe = 93883.11$, $p<.01$), and the interference effect of the vowel recognition task (278ms) was larger than that of the orientation recognition task (206ms). For the addition, on the other hand, the interference effect of the orientation recognition task (230ms) was marginally larger than that of the vowel decision task (198ms) ($F(1, 105) = 3.33$, $MSe = 18545.07$, $p<.1$). For the subtraction, the interference effect of the vowel recognition task (340ms) was not different from that of the orientation recognition task (350ms) significantly ($F(1, 105) = 0.35$, $MSe = 1947.25$, $p>.1$).

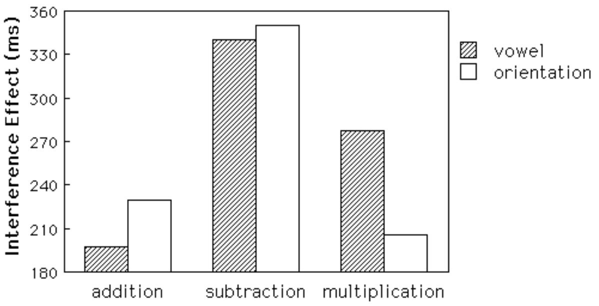

Figure 1: Interference effects of the vowel and the orientation recognition tasks on the mental addition, subtraction and multiplication

4. Discussion

The aim of the present study was to examine how the direct memory retrieval and the quantitative operation contribute to the mental addition, subtraction, and multiplication by using the dual-task procedure. For this aim, we assumed that the vowel and the orientation recognition were desirable as the secondary tasks because the vowel and the orientation recognition tasks included the similar subprocess and the previous studies indicated that these tasks could interfere with the direct memory retrieval or the quantitative operation, respectively (Dehaene, 1992; Fias et al., 2001; Rushworth et al., 1997).

Experiment 1 revealed that the interference effect of the vowel recognition task was larger than that of the orientation recognition task in the mental multiplication. In consistent with the previous studies, this result indicates that the mental multiplication depend more on the direct memory retrieval than on the quantitative operation (e.g., Ashcraft, 1995; Cohen & Dehaene, 2000; Lee & Kang, 2002).

On the contrary, the interference effect of the orientation recognition task trended to be larger than that of the vowel recognition task in the mental addition. As mentioned early, the findings of the addition are not consistent among studies (LeFevre, Sadesky & Bisanz, 1996; Miller, Perlmutter, & Keating, 1984). As a result, some researchers propose that the mental addition also entirely depends on the direct memory retrieval (e.g., Miller, Perlmutter, & Keating, 1984), whereas others claim that the quantitative operation (semantic elaboration) contributed the mental addition (e.g., LeFevre, Sadesky & Bisanz, 1996). The findings of the present study support to the latter one, indicating that the mental addition depends more on the quantitative operation than on the direct memory retrieval. The discrepancy in the findings about the mental addition might relate to the cultural factor (e.g., education system). In other words, different strategies for the mental addition might be employed in the different cultural people. In Japan, children are forced to memorize the multiplication table ("kuku": see Dehaene, 1997, p130). However, the addition and the subtraction tables are not learned as the multiplication. As the findings of the present study indicate, therefore, it is not surprising that the quantitative operation contributes to the solving of the addition more than that of the multiplication. Although the further studies are necessary to conclude the issue, at least, the finding of the present study and the other studies (e.g., LeFevre, Sadesky & Bisanz, 1996) revealed that the mental addition more or less depends on the quantitative operation. Hence, we propose that models of mental arithmetic in which utmost similarity (i.e., only direct memory retrieval) is assumed between the mental addition and multiplication should be revised (e.g., Ashcraft, 1992; 1995).

For the subtraction, the interference effect of the vowel recognition task was not different from that of the orientation recognition task. This finding is partly inconsistent with the findings from studies with brain damaged patients. For example, the result of Deahene and Cohen (1997) suggested that the mental subtraction depends on the quantitative operation (and Cohen & Dehaene, 2000). Rather, the finding of the present study indicates that the direct memory retrieval and the quantitative operation would contribute equally to the mental subtraction. It is possible that the property of the subtraction problems employed might relate to the different findings between the present and Dehaene's studies. In the present study, half of the first operands was smaller than the second operands (e.g., 2 5). Thus, half of the subtraction problems was negative answers. On the other hand, all of the subtraction problems was positive answers in Dehaene's studies. This difference might affect on the disparity of results between the present and Dehaene's studies. However, by brain-imaging technique (fMRI), Lee (2000) showed that intra-parietal sulci associate with the solving of the subtraction problem by using similar stimuli to the present study (i.e., 50 % negative problems). Because other brain imaging studies showed that the quantitative processing of numbers also associate with intra-parietal lobe (e.g, Dehaene et al., 1996), the negative problem might not critical factor which induce the different results between the present and the Dehaene's studies. To understand the mental process in the subtraction, further studies should be conducted. As the previous studies, however, the finding of the present study indicates that the mental subtraction depends less on the direct memory retrieval than the mental multiplication.

In conclusion, the mental multiplication must depend more on the direct memory retrieval than on the quantitative operation. On the other hand, the finding of the present study indicates that the mental addition might depend on the quantitative operation than on the direct memory retrieval. For the mental subtraction, the finding of the present study indicates that the direct memory retrieval and the quantitative operation contributed equally to the solving problems. Further studies must be conducted to understand the mental processing which are based on the simple mental arithmetic, especially the addition and subtraction.

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